Channel Aware Iterative Source Localization for Wireless Sensor Networks

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Abstract – *In this paper, we propose an energy efficient iterative source localization scheme in wireless sensor networks (WSNs). Instead of sending data from all the sensors to the fusion center, a coarse location estimate is first obtained from a set of anchor sensors. Then, a few non-anchor sensors are activated at a time to refine the location estimate in an iterative manner. We assume that the channels between sensors and the fusion center are subject to fading and noise. The fusion center is assumed to either have the complete or partial channel knowledge. Based on the received information at each iteration, the minimum mean squared error (MMSE) estimate of the source location is approximated using a Monte Carlo method. Then, in order to activate the non-anchor sensors for the next iteration, we develop a mutual information (MI)-based sensor selection scheme. Simulation results for the partial channel knowledge (PCK) and the complete channel knowledge (CCK) are presented to show the performance of the proposed approach.*

Keywords: Source localization, mutual information, posterior Cramér-Rao lower bound, sensor selection, wireless sensor networks.

1 Introduction

Wireless sensor networks (WSNs) are composed of a large number of densely deployed sensor nodes that cooperatively monitor the physical or environmental conditions of an event of interest such as temperature or velocity of an object. Recently, WSNs have found a wide range of application areas such as battlefield security, surveillance, environment or health monitoring, and disaster relief operations. In this paper, we study the source localization problem where the aim is to estimate the coordinates of a source that is emitting energy (e.g. acoustic source) [1], [2].

We have previously presented an iterative source localization method in [3], [4]. Rather than using a one-shot location estimator which requests the multi-bit decisions from all the sensors in the WSN, we proposed an iterative approach. A small number of anchor sensors are first employed to obtain a coarse location estimate. Then, a few non-anchor sensors are activated in an iterative manner to refine the location estimate. In this paper, we extend our method for the case where the channels between the sensors and the fusion center are subject to fading and noise.

For the source localization problem, in another previous work [5] imperfect channels were considered but the analysis was limited to 1-bit transmissions. In this work, we generalize the source location estimation approaches given in [2], [5] and consider M-bit sensor data. Assuming phase coherent reception, we consider the channel effects for two different scenarios. In the first case, we assume that complete channel information (CCK) of the channels between the sensors and the fusion center is available at the fusion center including the exact gain and phase information. In the second case, we assume that partial channel knowledge (PCK) is available where phase information and the statistics of the channel gain are known at the fusion center. We derive the posterior Cramér-Rao lower bound (PCRLB) of the source location estimate. In order to activate the non-anchor sensors at each iteration, we extend the mutual information (MI) based sensor selection scheme [6] to the case where quantized multi-bit data are transmitted from the sensors to the fusion center over fading channels.

The rest of the paper is organized as follows. In Section 2, we provide system assumptions and derive the posterior Cramér-Rao lower bound (PCRLB) of the estimate based on M-bit sensor measurements transmitted over fading channels. In Section 3, first we describe a Monte Carlo based method to obtain the MMSE estimate of the source location. We then derive the mutual information based sensor selection method under fading channels. In Section 4, we give some numerical examples to demonstrate the estimation performance and finally we devote Section 5 to our conclusions.

2 System Model

2.1 WSN assumptions

We consider a WSN consisting of N sensors $\{s_k, k = 1, 2, .., N\}$ and a fusion center. We assume that a signal (e.g., an acoustic signal) is radiated from a location with coordinates (x, y) and it follows an isotropic power attenuation model. In this paper, we assume that the source is based on flat ground and all the sensors and source have the same height so that a 2-D model is sufficient to formulate the problem. As an example, an acoustic event on the ground can be analyzed using a 2-D scenario as shown in Fig. 1. In this paper, we assume that N sensors are deployed in a grid layout and the WSN uses a parallel architecture where the quantized measurements of each sensor are directly delivered to the fusion center. The location of each sensor (s_k) is represented by (x_k, y_k). Then, the distance between s_k and the source location (x, y) is $d_k = \sqrt{(x - x_k)^2 + (y - y_k)^2}$. The received source energy a_k^2 at s_k is expressed as [2],

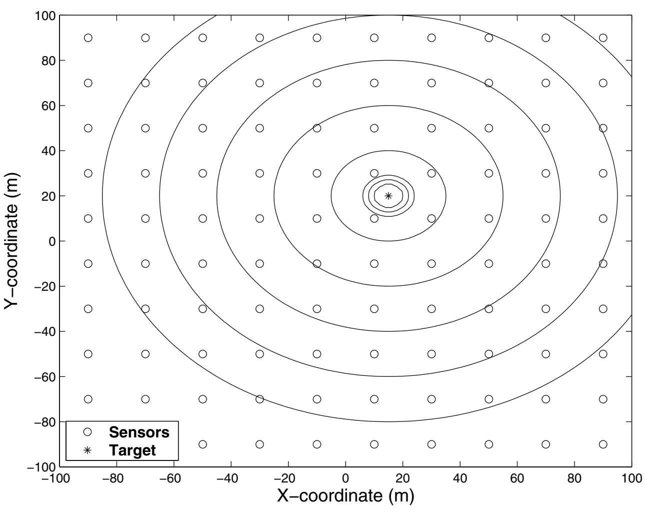

Figure 1: The signal intensity contours of a source located in a sensor field.

$$a_k^2 = P_0 \left(\frac{d_0}{d_k} \right)^n \tag{1}$$

where P_0 is the signal power measured at a reference distance d_0 (In this paper, we set $d_0 = 1m.$), a_k is the received signal amplitude at sensor s_k and n is the signal decay exponent. At each sensor, the received signal amplitude a_k is corrupted by an additive Gaussian noise:

$$z_k = a_k + \omega_k \tag{2}$$

where z_k is the noisy signal measurement at sensor s_k. Here, we assume that the observation noise ω_k is independent and identically distributed across sensors with Gaussian distribution $\mathcal{N}(0, \sigma_\omega^2)$, with $\sigma_\omega^2 = 1$.

We assume the same set of quantization thresholds at all the sensors $\boldsymbol{\eta} = [\eta_0, \eta_1, \ldots, \eta_L]^T$. Then D_k is obtained from z_k according to,

$$D_k = \begin{cases} 0 & -\infty < z_k < \eta_1 \\ 1 & \eta_1 < z_k < \eta_2 \\ \vdots & \\ L-1 & \eta_{L-1} < z_k < \infty \end{cases} \tag{3}$$

where $\eta_0 = -\infty$ and $\eta_L = \infty$.

Each local decision D_k is mapped to an M-bit binary sequence $\mathbf{B}_k$ as [7],

$$\mathbf{B}_k = [b_{k,1}, \ ..., b_{k,M}], \ (b_{k,j} \in \{0, 1\})$$

and transmitted using BPSK modulation as

$$\mathbf{Q}_k = [q_{k,1}, \ ..., q_{k,M}]$$

where

$$q_{k,j} = 2b_{k,j} - 1 \ , \ j \in \{1, 2, \ldots, M\} \ \ k \in \{1, 2, \ldots, N\}$$

We consider a discrete-time flat fading channel with a stationary and ergodic complex gain of $h_k e^{j\phi_k}$ between sensor k and the fusion center where h_k is the gain of the channel and ϕ_k is the phase of the channel. We assume that the channel remains constant during the transmission of $\mathbf{Q}_k$. The received symbols from s_k have the form,

$$\tilde{r}_{k,j} = \sqrt{\varepsilon_b} \ h_k \ e^{j\phi_k} \ q_{k,j} + \tilde{n}_{k,j} \tag{4}$$

where ε_b is the bit energy and $\tilde{n}_{k,j}$ is a zero-mean complex Gaussian noise with independent real and imaginary parts having identical variance σ_n^2. Then $\tilde{n}_{k,j} \sim \mathcal{CN}(0, 2\sigma_n^2)$. Let

$$\mathbf{R}_k = [r_{k,1}, r_{k,2}, \ ..., r_{k,M}]$$

be the soft-decoded symbols received from s_k after phase coherent reception [8]. Then $r_{k,j}$ has the form,

$$\begin{aligned} r_{k,j} &= \mathrm{Re}\left\{ \tilde{r}_{k,j} e^{-j\phi_k} \right\} \\ &= \sqrt{\varepsilon_b} \ h_k \ q_{k,j} + n_{k,j} \end{aligned} \tag{5}$$

where $n_{k,j} \sim \mathcal{N}(0, \sigma_n^2)$ is independent and identically distributed for each symbol.

In this paper, we assume two different cases for the channel. The first case assumes partial channel knowledge (PCK) where the phase information and the probability density function (pdf) of channel gain are known at the fusion center. The second case assumes complete channel knowledge (CCK) where both channel gain and phase information are available at the fusion center.

Given the source location, $\boldsymbol{\theta} = [x \ y]^T$, the likelihood $p(\mathbf{R}|\boldsymbol{\theta})$ becomes,

$$p(\mathbf{R}|\boldsymbol{\theta}) = \prod_{k=1}^{N} p(\mathbf{R}_k|\boldsymbol{\theta}) \tag{6}$$

In the above equation,

$$p(\mathbf{R}_k|\boldsymbol{\theta}) = \sum_{l=0}^{L-1} p(\mathbf{R}_k|D_k = l)p(D_k = l|\boldsymbol{\theta}) \quad (7)$$

where

$$p(\mathbf{R}_k|D_k = l) = \prod_{j=1}^{M} p(r_{k,j}|q_{k,j})$$

and

$$p(D_k = l|\boldsymbol{\theta}) = Q\left(\frac{\eta_l - a_k}{\sigma_\omega}\right) - Q\left(\frac{\eta_{l+1} - a_k}{\sigma_\omega}\right)$$

where $Q(.)$ is the complementary distribution function of the standard Gaussian distribution with zero mean and unit variance.

2.1.1 The likelihood of the received symbols under CCK

Under the independent channel noise assumption, the vector of symbols received from each sensor also becomes independent. We first assume that the complete channel knowledge is available at the receiver. Conditioning on channel gain h_k, we have,

$$p(\mathbf{R}_k|\mathbf{Q}_k, h_k) = \prod_{j=1}^{M} p(r_{k,j}|q_{k,j}, h_k) \quad (8)$$

where

$$p(r_{k,j}|q_{k,j}, h_k) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{(r_{k,j} - \sqrt{\varepsilon_b}\ h_k\ q_{k,j})^2}{2\sigma_n^2}\right)$$

Then (8) becomes,

$$p(\mathbf{R}_k|\mathbf{Q}_k, h_k) = \quad (9)$$

$$\frac{1}{(2\pi)^{M/2}\sigma_n^M} \exp\left(-\sum_{j=1}^{M} \frac{\left(r_{k,j} - \sqrt{\varepsilon_b} h_k q_{k,j}\right)^2}{2\sigma_n^2}\right)$$

2.1.2 The likelihood of the received symbols under PCK

We next incorporate imperfect channel statistics. Assuming a Rayleigh fading channel with unit power (i.e., $E[h_k^2] = 1$), the pdf of h_k is expressed as,

$$p(h_k) = 2h_k \exp\left(-h_k^2\right), \quad h_k \geqslant 0 \quad (10)$$

Now using (9), the likelihood of the received symbols at the fusion center is described as,

$$p(\mathbf{R}_k|\mathbf{Q}_k) = \int_{h_k} p(\mathbf{R}_k|\mathbf{Q}_k, h_k)p(h_k)dh_k \quad (11)$$

$$= \int_{h_k} \left(\prod_{j=1}^{M} p(r_{k,j}|h_k, q_{k,j})\right) p(h_k)dh_k$$

which is equivalent to,

$$p(\mathbf{R}_k|\mathbf{Q}_k) = \int_0^\infty \frac{2h_k \exp\left(-h_k^2\right)}{(2\pi)^{M/2}\sigma_n^M} \times \quad (12)$$

$$\exp\left(-\sum_{j=1}^{M} \frac{(r_{k,j} - \sqrt{\varepsilon_b}\ h_k\ q_{k,j})^2}{2\sigma_n^2}\right) dh_k$$

After some manipulations, (12) yields a closed form solution which is provided in the following proposition.

Proposition 1 *The conditional pdf of $\mathbf{R}_k$, given local decision D_k (or $\mathbf{Q}_k$) is*

$$p(\mathbf{R}_k|\mathbf{Q}_k) =$$

$$\frac{2}{(2\pi)^{M/2}\sigma_n^{M-2}\left(2\sigma_n^2 + \varepsilon_b\ M\right)} \exp\left(-\frac{\sum_{j=1}^{M} r_{k,j}^2}{2\sigma_n^2}\right) \times$$

$$\left[1 + \sqrt{2\pi}\beta\ \left(\sum_{j=1}^{M} r_{k,j}\ q_{k,j}\right)\right.$$

$$\exp\left(\frac{\beta^2\ \left(\sum_{j=1}^{M} r_{k,j}\ q_{k,j}\right)^2}{2}\right)$$

$$\left. Q\left(-\beta\ \left(\sum_{j=1}^{M} r_{k,j}\ q_{k,j}\right)\right)\right] \quad (13)$$

where

$$\beta = \frac{\sqrt{\varepsilon_b}}{\sigma_n\sqrt{2\sigma_n^2 + \varepsilon_b\ M}}$$

We omit the detailed proof of *Proposition 1.* Note that *Lemma 1* presented in [8] is a special case of (13) with $M = 1$.

2.2 PCRLB of the source location estimate

In this paper, we assume that the source location $\boldsymbol{\theta}$ follows a prior pdf $p_0(\boldsymbol{\theta})$ which is $\mathcal{N}(\boldsymbol{\mu_0}, \boldsymbol{\Sigma}_0)$ where $\boldsymbol{\mu_0}$ is the center of the ROI and $\boldsymbol{\Sigma}_0$ is the covariance matrix which is very coarse so that its confidence region covers the whole ROI. Let $p(\mathbf{R}, \boldsymbol{\theta})$ be the joint probability density of the pair $(\mathbf{R}, \boldsymbol{\theta})$. Then, the PCRLB of the estimation error has the form [9],

$$E\{[\hat{\boldsymbol{\theta}}(\mathbf{R}) - \boldsymbol{\theta})][\hat{\boldsymbol{\theta}}(\mathbf{R}) - \boldsymbol{\theta})]^T\} \geq \mathbf{J}_{\mathrm{PCRLB}}^{-1} \quad (14)$$

where $\mathbf{J}_{\mathrm{PCRLB}}$ is the 2×2 Fisher information matrix (FIM)

$$\mathbf{J}_{\mathrm{PCRLB}} = E\left[-\nabla_{\boldsymbol{\theta}}^{\boldsymbol{\theta}} \log p(\mathbf{R}, \boldsymbol{\theta})\right] \quad (15)$$

Using the equality $p(\mathbf{R}, \boldsymbol{\theta}) = p(\mathbf{R}|\boldsymbol{\theta})p_0(\boldsymbol{\theta})$, the Fisher information matrix (FIM) can be written as,

$$\mathbf{J}_{\text{PCRLB}} = E[-\nabla_{\boldsymbol{\theta}}^{\boldsymbol{\theta}} \log p(\mathbf{R}|\boldsymbol{\theta})] + E[-\nabla_{\boldsymbol{\theta}}^{\boldsymbol{\theta}} \log p_0(\boldsymbol{\theta})] = \mathbf{J}_d + \mathbf{J}_p \quad (16)$$

In (16), $\mathbf{J}_p \triangleq E[-\nabla_{\boldsymbol{\theta}}^{\boldsymbol{\theta}} \log p_0(\boldsymbol{\theta})] = \boldsymbol{\Sigma}_0^{-1}$ represents the *a priori* information, and $\mathbf{J_d} \triangleq E[-\nabla_{\boldsymbol{\theta}}^{\boldsymbol{\theta}} \log p(\mathbf{R}|\boldsymbol{\theta})]$ is the standard FIM averaged over the prior pdf of the source location as,

$$\mathbf{J}_d = E[-\nabla_{\boldsymbol{\theta}}^{\boldsymbol{\theta}} \log p(\mathbf{R}|\boldsymbol{\theta})] = \int_{\theta} \mathbf{J}(\boldsymbol{\theta}) p_0(\boldsymbol{\theta}) d\theta \quad (17)$$

where

$$\mathbf{J}(\boldsymbol{\theta}) = \sum_{k=1}^{N} \mathbf{J}_k(\boldsymbol{\theta}) \quad (18)$$

$\mathbf{J}_k(\boldsymbol{\theta})$ is the FIM of $\mathbf{R}_k$ and the first element of $\mathbf{J}_{k,11}(\boldsymbol{\theta})$ is calculated from,

$$\mathbf{J}_{k,11}(\boldsymbol{\theta}) = -E\left[\frac{\partial^2 \log p(\mathbf{R}_k|\boldsymbol{\theta})}{\partial x^2}\right] = \int_{\mathbf{R}_k} \frac{1}{p(\mathbf{R}_k|\boldsymbol{\theta})} \left(\frac{\partial p(\mathbf{R}_k|\boldsymbol{\theta})}{\partial x}\right)^2 d\mathbf{R}_k \quad (19)$$

where

$$\frac{\partial p(\mathbf{R}_k|\boldsymbol{\theta})}{\partial x} = \sum_{l=0}^{L-1} p(\mathbf{R}_k|D_k = l) \frac{\partial p(D_k = l|\boldsymbol{\theta})}{\partial x} \quad (20)$$

and

$$\frac{\partial p(D_k=l|\boldsymbol{\theta})}{\partial x} = \frac{n a_k (x - x_k)}{2\sqrt{2}\sigma_w d_k^2} \times \left[e^{-\frac{(\eta_l - a_k)^2}{2\sigma_w^2}} - e^{-\frac{(\eta_{l+1} - a_k)^2}{2\sigma_w^2}} \right] \quad (21)$$

Note that $\int_{\mathbf{R}_k}$ is an M-fold integration over the received symbols $(r_{k,1}, \ldots, r_{k,M})$. If CCK is available $p(\mathbf{R}_k|D_k = l, h_k)$ is calculated according to (9) otherwise $p(\mathbf{R}_k|D_k = l)$ is calculated according to (13). The other terms in (18) can be derived in a similar manner.

3 Iterative Source Location Estimation under Channel Fading

3.1 Sequential source localization using a Monte-Carlo method

At the beginning of the algorithm, the fusion center gathers M-bit information from each of the K anchor sensors and at each subsequent iteration of the algorithm, the fusion center gathers the M-bit data from additional A non-anchor sensors. Let $p(\boldsymbol{\theta}|\mathbf{W}_i)$ be the posterior pdf of the source location given the available data $\mathbf{W}_i = [\mathbf{R}_1,\ \mathbf{R}_2,\ ...,\ \mathbf{R}_{K+iA}]$ at iteration i $(i \geq 0)$. We then approximate $p(\boldsymbol{\theta}|\mathbf{W}_i)$ using a sequential importance sampling Monte-Carlo method as follows.

$$p(\boldsymbol{\theta}|\mathbf{W}_i) = \sum_{m=1}^{N_s} w^{m,i} \delta(\boldsymbol{\theta} - \boldsymbol{\theta}^{m,0}) \quad (22)$$

where $\boldsymbol{\theta}^{m,0}$ represents a particle at iteration i, which are drawn from the prior distribution $p_0(\boldsymbol{\theta})$ where $p_0(\boldsymbol{\theta}) \sim \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$ and $w^{m,i}$ represents the weight of a particle. Let $\tilde{w}^{m,i}$ be the weight of particle $\boldsymbol{\theta}^{m,0}$. For the first iteration $\tilde{w}^{m,1}$ is obtained as,

$$\tilde{w}^{m,1} \propto \prod_{k=1}^{K} p(\mathbf{R}_k|\boldsymbol{\theta}^{m,0}) w^{m,0} \quad (23)$$

where $\prod_{k=1}^{K} p(\mathbf{R}_k|\boldsymbol{\theta}^{m,0})$ is the likelihood of the K anchor sensor data at the end of the first iteration. For the rest of the iterations $(i > 1)$,

$$\tilde{w}^{m,i} \propto \prod_{k=1}^{A} p(\mathbf{R}_k|\boldsymbol{\theta}^{m,0}) \tilde{w}^{m.i-1} \quad (24)$$

Note that $\prod_{k=1}^{A} p(\mathbf{R}_k|\boldsymbol{\theta}^{m,0})$ is the likelihood of the A non-anchor sensors activated at each iteration. The particle weights are further normalized as,

$$w^{m,i} = \frac{\tilde{w}^{m,i}}{\sum_{m=1}^{N_s} \tilde{w}^{m,i}} \quad (25)$$

Then the MMSE of the source location can be approximated as,

$$\hat{\boldsymbol{\theta}}_i^{MMSE} = \sum_{m=1}^{N_s} w^{m,i} \boldsymbol{\theta}^{m,0} \quad (26)$$

3.2 Mutual Information Based Sensor Selection under Channel Fading

Let $\mathcal{R}_A^i = \left\{\mathcal{R}_A^{(i,1)}, \mathcal{R}_A^{(i,2)},\ ...,\ \mathcal{R}_A^{(i,C_A^{N-K-iA})}\right\}$ be the collection of all distinct A-element subsets of $N-K-iA$ remaining non-anchor sensors for the iteration i where C_A^{N-K-iA} is the combination operation.

Let $\mathcal{R}_A^{(i,\nu)}$ be the set of A non-anchor sensors to be activated at the i^{th} iteration according to the sensor selection strategy ν. Then, $\mathcal{R}_A^{(i,\nu)} = \left\{\mathbf{R}_1^{i,\nu}, \mathbf{R}_2^{i,\nu},\ ...,\ \mathbf{R}_{\mathbf{A}}^{i,\nu}\right\}$ and $\mathbf{R}_k^{i,\nu}$ $(k \in \{1, .., A\})$ are the received symbols from the k^{th} activated non-anchor sensor according to ν at iteration i. Then, $\mathbf{R}_k^{i,\nu} = [r_{k,1}^{i,\nu}, ..., r_{k,M}^{i,\nu}]$. Now, the objective is to find the optimal sensor selection strategy ν^* which activates the A non-anchor sensors $\mathcal{R}_A^{(i,\nu^*)} = \{\mathbf{R}_1^{i,\nu^*},\ ..., \mathbf{R}_A^{i,\nu^*}\}$ whose data minimize the expected conditional entropy of the posterior source location distribution as,

$$\nu^* = \arg\min_{\nu} H(\boldsymbol{\theta}|\mathcal{R}_A^{(i,\nu)}) \quad (27)$$

Let $I(\boldsymbol{\theta}, \mathcal{R}_A^{(i,\nu)}) = H(\boldsymbol{\theta}) - H(\boldsymbol{\theta}|\mathcal{R}_A^{(i,\nu)})$ be the mutual information between the source location $\boldsymbol{\theta}$ and the measurements of the activated sensors according to the activation scheme ν. The sensor selection problem now turns into,

$$\nu^* = \arg\max_{\nu} I(\boldsymbol{\theta}, \mathcal{R}_A^{(i,\nu)}) \tag{28}$$

$I(\boldsymbol{\theta}, \mathcal{R}_A^{(i,\nu)})$ can also be expanded as [6],

$$I(\boldsymbol{\theta}, \mathcal{R}_A^{(i,\nu)}) = H(\mathcal{R}_A^{(i,\nu)}) - H(\mathcal{R}_A^{(i,\nu)}|\boldsymbol{\theta}) \tag{29}$$

To compute (29) using Monte-Carlo approximation, we start with writing the entropy of $\mathcal{R}_A^{(i,\nu)}$,

$$H(\mathcal{R}_A^{(i,\nu)}) = -\int p(\mathcal{R}_A^{(i,\nu)}) \log p(\mathcal{R}_A^{(i,\nu)}) \tag{30}$$

where given source location $\boldsymbol{\theta}$, $p(\mathcal{R}_A^{(i,\nu)})$ can be decomposed as,

$$p(\mathcal{R}_A^{(i,\nu)}) = \int_{\theta} p(\mathbf{R}_1^{i,\nu}|\boldsymbol{\theta}) \, ...p(\mathbf{R}_A^{i,\nu}|\boldsymbol{\theta}) p_i(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{31}$$

Note that for iteration i, $p(\boldsymbol{\theta}|\mathbf{W}_{i-1})$ serves as the prior pdf of the source location $p_i(\boldsymbol{\theta})$. Then $p_i(\boldsymbol{\theta}) = p(\boldsymbol{\theta}|\mathbf{W}_{i-1})$ which is obtained by the previously presented importance sampling-based Monte Carlo method and $p(\mathbf{R}_1^{i,\nu}|\boldsymbol{\theta})$, ..., $p(\mathbf{R}_A^{i,\nu}|\boldsymbol{\theta})$ are the likelihood functions. Using (22), (31) becomes,

$$p(\mathcal{R}_A^{(i,\nu)}) = \sum_{m=1}^{N_s} w^{m,i-1} p(\mathbf{R}_1^{i,\nu}|\boldsymbol{\theta}^{m,0}) \, ...p(\mathbf{R}_A^{i,\nu}|\boldsymbol{\theta}^{m,0}) \tag{32}$$

With (32), $H(\mathcal{R}_A^{(i,\nu)})$ defined in (30) is rewritten as follows,

$$\begin{aligned} &H(\mathcal{R}_A^{(i,\nu)}) = \\ &-\int_{\mathbf{R}_1^{i,\nu},...,\mathbf{R}_A^{i,\nu}} \left\{ \left[\sum_{m=1}^{N_s} w^{m,i-1} \prod_{k=1}^{A} p(\mathbf{R}_k^{i,\nu}|\boldsymbol{\theta}^{m,0}) \right] \right. \\ &\left. \times \log \left[\sum_{m=1}^{N_s} w^{m,i-1} \left(\prod_{k=1}^{A} p(\mathbf{R}_k^{i,\nu}|\boldsymbol{\theta}^{m,0}) \right) \right] \right\} \\ &d\mathbf{R}_1^{i,\nu} \, ...d\mathbf{R}_A^{i,\nu} \end{aligned} \tag{33}$$

Now let us compute the second term of (29). First we have,

$$\begin{aligned} &H(\mathcal{R}_A^{(i,\nu)}|\boldsymbol{\theta}) = \\ &-\int_{\theta} \int_{\mathbf{R}_1^{i,\nu}...\mathbf{R}_A^{i,\nu}} p(\mathcal{R}_A^{(i,\nu)}, \boldsymbol{\theta}) \log p(\mathcal{R}_A^{(i,\nu)}|\boldsymbol{\theta}) \\ &d\mathbf{R}_1^{i,\nu}...d\mathbf{R}_A^{i,\nu} d\boldsymbol{\theta} \end{aligned} \tag{34}$$

Since $p(\mathcal{R}_A^{(i,\nu)}, \boldsymbol{\theta}) = p(\mathcal{R}_A^{(i,\nu)}|\boldsymbol{\theta}) p_i(\boldsymbol{\theta})$ and considering the Monte-Carlo approximation of the prior source location pdf (22), (34) is expressed as,

$$\begin{aligned} &H(\mathcal{R}_A^{(i,\nu)}|\boldsymbol{\theta}) = -\sum_{k=1}^{A} \int_{\mathbf{R}_k^{i,\nu}} \\ &\left[\sum_{m=1}^{N_s} w^{m,i-1} p(\mathbf{R}_k^{i,\nu}|\boldsymbol{\theta}^{m,0}) \log(p(\mathbf{R}_k^{i,\nu}|\boldsymbol{\theta}^{m,0})) d\mathbf{R}_k^{i,\nu} \right] \end{aligned} \tag{35}$$

Finally using (33) and (35), the mutual information $I(\boldsymbol{\theta}, \mathcal{R}_A^{(i,\nu)})$ expressed in (29) is calculated as follows.

$$\begin{aligned} &I(\mathcal{R}_A^{(i,\nu)}, \boldsymbol{\theta}) = \\ &-\int_{\mathbf{R}_1^{i,\nu}} ... \int_{\mathbf{R}_A^{i,\nu}} \left[\sum_{m=1}^{N_s} w^{m,i-1} \left(\prod_{k=1}^{A} p(\mathbf{R}_k^{i,\nu}|\boldsymbol{\theta}^{m,0}) \right) \right] \\ &\log \left[\sum_{m=1}^{N_s} w^{m,i-1} \left(\prod_{k=1}^{A} p(\mathbf{R}_k^{i,\nu}|\boldsymbol{\theta}^{m,0}) \right) \right] d\mathbf{R}_1^{i,\nu} \, ...d\mathbf{R}_A^{i,\nu} \\ &+\sum_{k=1}^{A} \int_{\mathbf{R}_k^{i,\nu}} \left[\sum_{m=1}^{N_s} w^{m,i-1} p(\mathbf{R}_k^{i,\nu}|\boldsymbol{\theta}^{m,0}) \right. \\ &\left. \log(p(\mathbf{R}_k^{i,\nu}|\boldsymbol{\theta}^{m,0})) d\mathbf{R}_k^{i,\nu} \right] \end{aligned} \tag{36}$$

4 Simulation Results

In our examples, we consider the source energy and signal decay exponent as $P_0 = 2500$ and $n = 2$ respectively. N sensors are deployed in a $20 \times 20m^2$ field in a grid layout and we use $M = 3$ bits to quantize analog measurements. The decision thresholds of each sensor $\boldsymbol{\eta}$ is selected according to the method described in [2]. We consider two different scenarios which are the low channel signal-to-noise ratio (SNR) and high channel SNR cases with $\varepsilon_b = 1$ and $\varepsilon_b = 5$ respectively. The mean squared error (MSE) matrix of the estimation is calculated according to,

$$\text{MSE} = \frac{1}{N_{MC}} \sum_{u=1}^{N_{MC}} (\hat{\boldsymbol{\theta}}_u - \boldsymbol{\theta}_u)(\hat{\boldsymbol{\theta}}_u - \boldsymbol{\theta}_u)^T \tag{37}$$

where the MSE is obtained over N_{MC} Monte Carlo trials. The integrations with respect to $\mathbf{R}_k$ are performed by Monte Carlo integration [10]. The parameters of the prior probability distribution of the source location $p_0(.)$ is chosen such that $\boldsymbol{\mu}_0 = [10 \ m. \ 10 \ m.]^T$ and $\boldsymbol{\Sigma}_0 = \begin{bmatrix} \sigma_{x,0}^2 & 0 \\ 0 & \sigma_{y,0}^2 \end{bmatrix}$ is very coarse and its 99% confidence region covers the whole ROI.

4.1 Performance of the One-Shot Location Estimation

Let us consider one shot location estimation and find the Bayesian estimate of the source location using all sensor data. In one-shot location estimation, the fusion center receives data from all N sensors in the network and the weights of particles are updated using N sensor data. The MMSE of the estimate is then computed according to (26). The location estimate is averaged over $N_{\rm MC} = 1000$ different trials. We plot the results for the trace of the MSE and PCRLB matrices in Fig. 2. We compare the MSE performance of the estimate with its PCRLB bound. As we can see, even at low channel SNR, the MSE performance of PCK is very close to the CCK case. As the bit energy increases, the MSE performances of CCK and PCK cases are almost indistinguishable. Moreover, under high channel SNR and large number of sensors, the MSE performance of PCK case gets very close to its PCRLB bound.

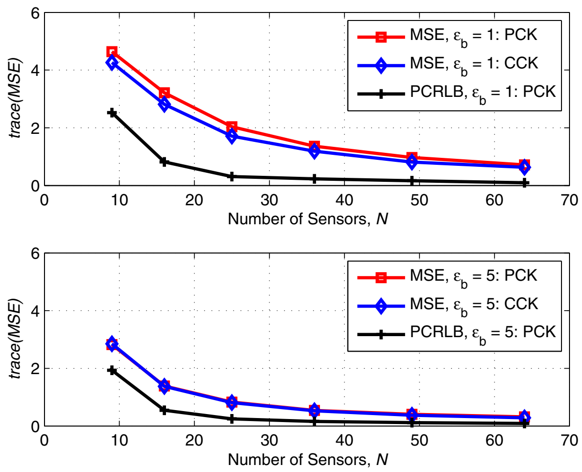

Figure 2: MSE of N sensor data, $(M = 3)$.

4.2 Performance of the Iterative Location Estimation

For the proposed iterative source localization algorithm, we employ $N = 7 \times 7 = 49$ sensors deployed in a grid layout. The algorithm is initialized with $K = 3 \times 3 = 9$ anchor sensors. The MSE of the estimate at each iteration is averaged over $N_{MC} = 500$ different trials. In our simulations, we activate $A = 1$ sensor at a time after the initialization via anchor sensors. As shown in Figure 3, for low channel SNR, CCK yields better performance than PCK as a result of using the complete channel information. Increasing the bit energy yields similar estimation performance for CCK and PCK. At high channel SNR, the MSE of iterative source localization algorithm converges to the MSE of all sensor data in about 20 iterations that is when the sensors close to source location are selected. On the other hand, at low channel SNR, the sensors become less informative, hence about 30 sensors should be selected to achieve the MSE of all sensor data. The mean channel gain and the mean distance to the source location of sensors selected at each iteration are shown in Figure 4. Under low channel SNR, MI-based sensor selection activates the sensors with large channel gains where the sensors are within a radius of 7 meters of the source location. As the channel SNR increases, the channel gain becomes less important and sensors in a closer proximity of the source location are selected.

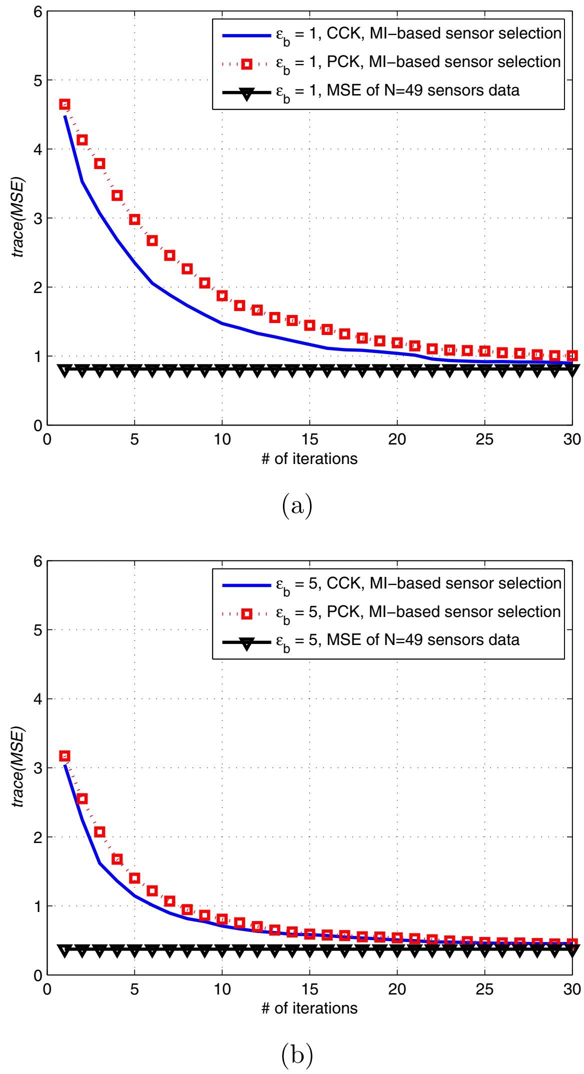

Figure 3: MSE performance of the iterative scheme using MI-based sensor selection (a) $\varepsilon_b = 1$, (b) $\varepsilon_b = 5$, $(M = 3)$.

5 Conclusions

In this paper, we have studied the iterative source localization problem where the multi-bit data of sen-

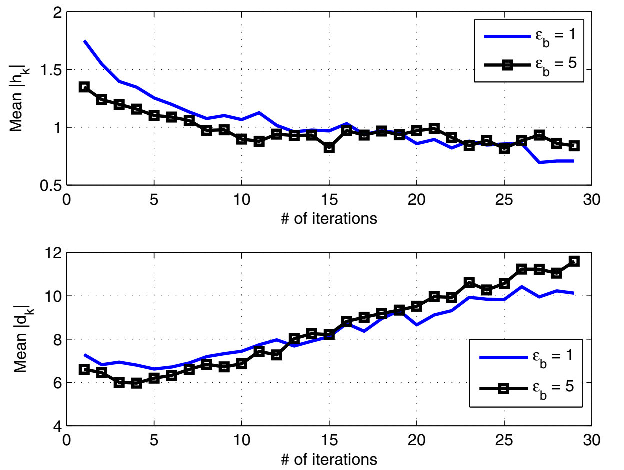

Figure 4: Mean channel gain $|h_k|$; mean distance between the source location and the selected sensor $|d_k|$ at each iteration.

sors are transmitted over fading channels. For multi-bit sensor data, we have derived the PCRLB of the source location estimate under channel fading. We have shown that having partial channel knowledge provides estimation performance very close to the case where complete channel information is available. We have also shown that, for high channel SNR and large number of sensors, the MSE is very close to its PCRLB bound. For the iterative source localization scheme, we have derived the MI based sensor selection scheme. Simulation results show that under CCK and low channel SNR, the MI-based sensor selection scheme gives more priority to the sensors with large channel gain. As channel SNR increases, the PCK assumption yields a similar iterative estimation performance to the CCK assumption.

Note that the computational complexity of the MI-based sensor selection increases exponentially with the number of sensors to be selected [4]. Future work will include developing new sensor selection metrics which provide good estimation performance with less computational complexity. Extension of our methodology for non-coherent reception for multi-bit sensor data will also be addressed.

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